NAME (PRINT): ____

Last / Surname First / Given Name

STUDENT #: ____________________ **TUTORIAL:** ____________________

MAT 137Y - 2018/2019
TEST 2 - VERSION A

Problem	1	2	3	4	5	Total
Points	10	10	10	10	10	50
Score						

Tutorial Sections:

◯ Jayde Silva, TUT0101 (TUE @ 1).
◯ Jayde Silva, TUT0103 (TUE @ 3).
◯ Jayde Silva, TUT0105 (TH @ 1).
◯ David Scott, TUT0108 (TH @ 3).
◯ Rodney Louis, TUT0102 (TUE @ 1).
◯ Rodney Louis, TUT0104 (TUE @ 3).
◯ Kenny Guo, TUT0107 (TH @ 3).

INSTRUCTIONS:

- **Please make sure your name, student number, and tutorial info is entered *in ink* at the top of this page.**
- **There is a 1 point penalty for missing, or incorrect TUT information.**
- **Do not begin until instructed to do so.**
- **You have 100 minutes to complete this test.**
- **Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided. If you choose to write in pencil, you will not be eligible for a re-grade.**
- **You may use Page 9 for rough work. Your rough work will not be graded.**
- **This test contains 9 pages. Please ensure they are all there.**
- **Please do not tear out any pages.**
- **No aids are allowed.**

GOOD LUCK!

Answer the following questions in the space provided. Please give complete, detailed solutions. Correct final answers with little, or no, work, will not receive credit.

(1) (a) (2pts) Let $y = \sin^{-1}(\cos x)$. Find y'', simplifying your answer as much as possible.

$y'' =$ ____________________________________.

(b) (2pts) Let $f(x) = \dfrac{1}{x^2 - x}$. Find $f'(e^x)$.

$f'(e^x) =$ ____________________________________.

Continues on the next page →

(c) (3pts) Consider the curve given by $x^2y^3 + \cos(xy) = xe^{2y}$. Find the equation of the tangent line at $(1, 0)$.

The equation of the tangent line is: ____.

(d) (3pts) Find all critical points of the function $h(t) = \sin t + \cos t$.

The critical points are at $x =$ ____.

Continues on the next page →

(2) Suppose that f is differentiable for all $x \in \mathbb{R}$ and that $f(0) = 0$. Consider the function $g(x) = f(x)|x|$.

(a) (8pts) Prove that g is differentiable at $x = 0$.

(b) (2pts) Prove that g is differentiable everywhere.

Continues on the next page →

(3) Suppose that $f : [-1, 1] \to [-1, 1]$ is continuous. Prove that the equation $|f(x)| = |x|$ has a solution for some $x \in [-1, 1]$.

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(4) Emery is 6 feet tall and walks at a constant rate of 3 ft/s along a straight path on flat ground. There is a light at the top of an 18 foot tall lamppost, located 4 feet to the side of the path. How fast is the length of Emery's shadow increasing, when she is 3 feet past the point on the path directly across from the lamppost?

(Hint: Draw a picture from above ***and*** the side.)

Continues on the next page →

(5) Determine if the statements below are true or false. If true, give a detailed explanation why using definitions, properties, or theorems from class. (An example will not suffice.) If false, explain why, or you may simply give an example which demonstrates why it's false. (You will receive no credit for simply guessing "true" or "false".)

(a) If f is bounded on $[a, b]$, then f has a global maximum on $[a, b]$.

TRUE FALSE

(b) If $h(x) = e^x g(x)$ is differentiable for all $x \in \mathbb{R}$, then g is differentiable for all $x \in \mathbb{R}$.

TRUE FALSE

(c) If f^3 is continuous for all $x \in \mathbb{R}$, then f is continuous for all $x \in \mathbb{R}$.

TRUE FALSE

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(d) If f, g are differentiable for all $x \in \mathbb{R}$, then the function $M(x) = \max\{f(x), g(x)\}$ is differentiable for all $x \in \mathbb{R}$.

TRUE FALSE

(e) If f, g each have a local maximum at $x = c$, then fg has a local maximum at $x = c$.

TRUE FALSE

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Rough Work - This page will not be graded!

The End.