MAT135H Term Test No.3 Questions

1 Question No.2 - Easy (2 points)

Version 1

The slope m_{tan} of the tangent line to the curve $y = g(x)$ at the point $R(x_0, g(x_0))$ is given by

$$m_{\text{tan}} = \lim_{x \to x_0} \rule{2em}{0.4pt} = \lim_{h \to 0} \rule{2em}{0.4pt}$$

Fill in the two blanks. You must clearly and coherently write your final answer with the appropriate limit in front of your final answer. Circle your final answers.

Version 2

The slope m_{tan} of the tangent line to the curve $y = k(x)$ at the point $Q(x_0, k(x_0))$ is given by

$$m_{\text{tan}} = \lim_{x \to x_0} \rule{2em}{0.4pt} = \lim_{h \to 0} \rule{2em}{0.4pt}$$

Fill in the two blanks. You must clearly and coherently write your final answer with the appropriate limit in front of your final answer. Circle your final answers.

Version 3

The slope m_{tan} of the tangent line to the curve $y = n(x)$ at the point $U(x_0, n(x_0))$ is given by

$$m_{\text{tan}} = \lim_{x \to x_0} \rule{2em}{0.4pt} = \lim_{h \to 0} \rule{2em}{0.4pt}$$

Fill in the two blanks. You must clearly and coherently write your final answer with the appropriate limit in front of your final answer. Circle your final answers.

Version 4

The slope m_{tan} of the tangent line to the curve $y = s(x)$ at the point $T(x_0, s(x_0))$ is given by

$$m_{\text{tan}} = \lim_{x \to x_0} \rule{2em}{0.4pt} = \lim_{h \to 0} \rule{2em}{0.4pt}$$

Fill in the two blanks. You must clearly and coherently write your final answer with the appropriate limit in front of your final answer. Circle your final answers.

2 Question No.3 - Medium (3 points)

Version 1

Find an equation of the normal line to the graph of $y = u(x)$ at $x = -4$, if $u(-4) = 5$ and $u'(-4) = -3$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Your final answer should be simplified to $y = mx + b$ form. Circle your final answer.

Version 2

Find an equation of the normal line to the graph of $y = g(x)$ at $x = -3$, if $g(-3) = 11$ and $g'(-3) = -2$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Your final answer should be simplified to $y = mx + b$ form. Circle your final answer.

Version 3

Find an equation of the normal line to the graph of $y = k(x)$ at $x = 4$, if $k(4) = -5$ and $k'(4) = 5$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Your final answer should be simplified to $y = mx + b$ form. Circle your final answer.

Version 4

Find an equation of the normal line to the graph of $y = n(x)$ at $x = -1$, if $n(-1) = -3$ and $n'(-1) = 10$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Your final answer should be simplified to $y = mx + b$ form. Circle your final answer.

3 Question No.4 - Medium (3 points - 1.5 points for each part)

Version 1

Find $F'(\pi)$ given that $f(\pi) = 1$, $f'(\pi) = -2$, $g(\pi) = 2$, and $g'(\pi) = -1$.

Part A: $F(x) = x^2(4f(x) - 7g(x))$

Part B: $F(x) = \dfrac{xf(x)}{7x + 2g(x)}$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for Part A and Part B.

Version 2

Find $G'(\pi)$ given that $f(\pi) = -2$, $f'(\pi) = 1$, $g(\pi) = -1$, and $g'(\pi) = 2$.

Part A: $G(x) = 5x(3f(x) - 8g(x))$

Part B: $G(x) = \dfrac{2f(x)}{x^2 + 2g(x)}$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for Part A and Part B.

Version 3

Find $H'(\pi)$ given that $s(\pi) = 1$, $s'(\pi) = -1$, $t(\pi) = -2$, and $t'(\pi) = 2$.

Part A: $H(x) = 5x(3t(x) - 8s(x))$

Part B: $H(x) = \dfrac{2s(x)}{x^2 + 2t(x)}$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for Part A and Part B.

Version 4

Find $H'(\pi)$ given that $s(\pi) = 2$, $s'(\pi) = -2$, $t(\pi) = -1$, and $t'(\pi) = 1$.

Part A: $H(x) = x^2(4t(x) - 7s(x))$

Part B: $H(x) = \dfrac{xs(x)}{7x + 2t(x)}$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for Part A and Part B.

4 Question No.5 - Difficult (4 points)

Version 1 & 2

Let $f(x) = \dfrac{-1}{\sqrt{3-x}}$. Using the definition of the derivative, find $f'(1)$.

You cannot use differentiation techniques.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 3 & 4

Let $f(x) = \dfrac{1}{\sqrt{4-x}}$. Using the definition of the derivative, find $f'(2)$.

You cannot use differentiation techniques.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

5 Question No.6 - Challenging (4 points)

Version 1 & 4

Suppose that

$$f(x) = \begin{cases} 5x + 2x^4 \cos\left(\dfrac{5}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Is $f(x)$ differentiable at $x = 0$?

Hint: Squeeze theorem.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 2 & 3

Suppose that

$$f(x) = \begin{cases} -3x + 2x^4 \sin\left(\dfrac{4}{x^2}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Is $f(x)$ differentiable at $x = 0$?

Hint: Squeeze theorem.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.