MAT135H Term Test No.2 Questions

1 Question No.1 - Easy (1 point for each function)

Version 1

Find $\lim_{x \to 0} f(x)$, where $x + 1 \leq f(x) \leq e^x$ for all x in the interval $(-1, 1)$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal numbers as a final answer.

Version 2

Find $\lim_{x \to 1} h(x)$, where $1 - 2x \leq h(x) \leq -x^2$ for all x in the interval $(0, 2)$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal number as a final answer.

Version 3

Find $\lim_{x \to 0} h(x)$, where $e^{-x} \leq h(x) \leq -x + 1$ for all x in the interval $(-1, 1)$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal number as a final answer.

Version 4

Find $\lim_{x \to 1} f(x)$, where $2x - 1 \leq f(x) \leq x^2$ for all x in the interval $(0, 2)$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal number as a final answer.

2 Question No.2 - Medium (3 points - 1.5 points for each part)

Version 1

Assume that $f(x)$ is a continuous function for all real numbers. Determine the following limits if $\lim_{x \to 0^+} f(x) = A$ and $\lim_{x \to 0^-} f(x) = B$:

Part A: $\lim_{x \to 0^-} \Big(f(x^3) - f(x)\Big)$

Part B: $\lim_{x \to 0^-} f(x^2 - x)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 2

Assume that $g(x)$ is a continuous function for all real numbers. Determine the following limits if $\lim_{x \to 0^+} g(x) = C$ and $\lim_{x \to 0^-} g(x) = D$:

Part A: $\lim_{x \to 0^+} \Big(g(x^2) - g(x)\Big)$

Part B: $\lim_{x \to 0^+} g(x^3 - x)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 3

Assume that $f(x)$ is a continuous function for all real numbers. Determine the following limits if $\lim_{x \to 0^+} f(x) = C$ and $\lim_{x \to 0^-} f(x) = D$:

Part A: $\lim_{x \to 0^-} \Big(f(x^3) - f(x)\Big)$

Part B: $\lim_{x \to 0^-} f(x^2 - x)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 4

Assume that $g(x)$ is a continuous function for all real numbers. Determine the following limits if $\lim_{x \to 0^+} g(x) = A$ and $\lim_{x \to 0^-} g(x) = B$:

Part A: $\lim_{x \to 0^+} \Big(g(x^2) - g(x)\Big)$

Part B: $\lim_{x \to 0^+} g(x^3 - x)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

3 Question No.3 - Medium (3 points - 1 point for each part)

Version 1

Find all horizontal asymptotes, if any, of the function $f(x) = \sqrt{x}(\sqrt{x+3} - \sqrt{x-2})$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 2

Find all horizontal asymptotes, if any, of the function $f(x) = \sqrt{x}(\sqrt{2x+1} - \sqrt{2x-1})$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 3

Find all horizontal asymptotes, if any, of the function $f(x) = \sqrt{x}(\sqrt{3x+7} - \sqrt{3x-7})$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 4

Find all horizontal asymptotes, if any, of the function $f(x) = \sqrt{x}(\sqrt{x+5} - \sqrt{x-2})$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

4 Question No.4 - Difficult (4 points)

Version 1

For what values of a and b is the following function continuous for all real numbers?

$$g(x) = \begin{cases} b - \dfrac{a(x+1)}{\mid 1-x \mid} & \text{for } x \leq -1 \\ 4\arctan(x) + a & \text{for } -1 < x < 1 \\ 3be^x - a\ln(x) & \text{for } x \geq 1 \end{cases}$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for a and b.

Version 2

For what values of a and b is the following function continuous for all real numbers?

$$f(x) = \begin{cases} \dfrac{a\cos(x)}{3} + b - 2 & \text{for } x < 0 \\ a + \dfrac{b \mid x-2 \mid}{x-2} & \text{for } 0 \leq x < 2 \\ 3x^2 - 7b & \text{for } x \geq 2 \end{cases}$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for a and b.

Version 3

For what values of m and n is the following function continuous for all real numbers?

$$f(x) = \begin{cases} -x^2 + 3 & \text{for } x < -1 \\ 2n + \dfrac{m(x+1)}{\mid x+1 \mid} & \text{for } -1 \leq x < 0 \\ -\dfrac{m\cos(x)}{2} + n + 1 & \text{for } x \geq 0 \end{cases}$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for m and n.

Version 4

For what values of m and n is the following function continuous for all real numbers?

$$g(x) = \begin{cases} m + \dfrac{n(x-3)}{\mid 3-x \mid} & \text{for } x < 0 \\ -4m\arctan(x) + ne^{2x} & \text{for } 0 \leq x \leq 1 \\ 3n + m\ln(x) + 1 & \text{for } x > 1 \end{cases}$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for m and n.

5 Question No.5 - Challenging (4 points)

Version 1

Show that the equation $4^x = \dfrac{34}{x}$ has a solution for $x > 0$.

Make sure to verify all assumptions.

You must clearly and coherently justify your work.

Version 2

Show that the equation $3^x = \dfrac{20}{x}$ has a solution for $x > 0$.

Make sure to verify all assumptions.

You must clearly and coherently justify your work.

Version 3

Show that the equation $5^x = \dfrac{60}{x}$ has a solution for $x > 0$.

Make sure to verify all assumptions.

You must clearly and coherently justify your work.

Version 4

Show that the equation $2^x = \dfrac{10}{x}$ has a solution for $x > 0$.

Make sure to verify all assumptions.

You must clearly and coherently justify your work. You cannot provide only the final answer.