

MAT135H Term Test No.1 Questions

1 Question No.1 - Easy (2 points)

Version 1 Let $f(x) = 2x^3 + 5x^2 + 5x + 1$. Find x if $f^{-1}(x) = 1$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal numbers as a final answer.

Version 2

Let $f(x) = \frac{x^3 + 1}{x^2 - 5} + e^{-3x}$. Find x if $f^{-1}(x) = 0$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal number as a final answer.

Version 3

Let $f(x) = 3x^3 + \ln(5x^2 - 4)$. Find x if $f^{-1}(x) = 1$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal number as a final answer.

Version 4

Let $f(x) = x \sin(3x) + 2 \cos(x)$. Find x if $f^{-1}(x) = \pi$.

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer. No decimal number as a final answer.

2 Question No.2 - Medium (3 points)

Given the inverse trigonometric identity:

$$\arctan(x) + \arctan(y) = \arctan\left(\frac{x+y}{1-xy}\right),$$

where $-\frac{\pi}{2} \leq \arctan(x) + \arctan(y) \leq \frac{\pi}{2}$.

Version 1

Does the following equation hold true? That is, the left-hand side of the equation equals the right-hand side.

$$2 \arctan\left(\frac{1}{3}\right) + \arctan\left(\frac{1}{7}\right) = \frac{\pi}{4}$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

Version 2

Does the following equation hold true? That is, the left-hand side of the equation equals the right-hand side.

$$\arctan\left(\frac{1}{5}\right) + 2 \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer.

3 Question No.3 - Medium (3 points)

Version 1

Let $f(x) = 1/x$, $g(x) = x^2$, and $h(x) = x - 3$. Find the value of each composition, or else say that it is not defined:

Part (A) $(f \circ (g \circ h - f))(1)$

Part (B) $((f + g) \circ (h - f))(1)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each part. No decimal number as a final answer.

Version 2

Let $f(x) = 1/x$, $g(x) = x^2$, and $h(x) = x - 3$. Find the value of each composition, or else say that it is not defined:

Part (A) $(f \circ (f \circ g - h))(1)$

Part (B) $((h + g) \circ (f \circ g))(1)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each part. No decimal number as a final answer.

Version 3

Let $f(x) = 1/x^2$, $g(x) = x$, and $h(x) = x + 5$. Find the value of each composition, or else say that it is not defined:

Part (A) $(f \circ (f \circ g - h))(1)$

Part (B) $((f + g) \circ (h - f))(1)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each part. No decimal number as a final answer.

Version 4

Let $f(x) = 1/x^2$, $g(x) = x$, and $h(x) = x + 5$. Find the value of each composition, or else say that it is not defined:

Part (A) $(g \circ (g \circ f - h))(1)$

Part (B) $((h + g) \circ (f \circ g))(3)$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each part. No decimal number as a final answer.

4 Question No.4 - Difficult (4 points - each function is 2 points)

Version 1

Find the domain of each function, and write the domain in interval notation.

$$k(x) = \frac{2x - \sqrt{x^2 + 2}}{\sqrt{16 - x^2}}$$

$$c(x) = 2 - \arcsin\left(\frac{3\pi x - 1}{2} - 5\right) + x$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each function. No decimal number as a final answer.

Version 2

Find the domain of each function, and write the domain in interval notation.

$$n(x) = \frac{e^{x^2-3} + 5x}{|64 - x^2|}$$

$$g(x) = 3 \arccos\left(\frac{1 - 4\pi x}{4} + 5\right) - \pi x$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each function. No decimal number as a final answer.

Version 3

Find the domain of each function, and write the domain in interval notation.

$$h(x) = \frac{\ln(-x^2 - 6x - 8)}{x^2 + 2}$$

$$m(x) = 2\sqrt{5 - 2x} - \arctan\left(\frac{3 - \pi x}{2} + 2\right)$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each function. No decimal number as a final answer.

Version 4

Find the domain of each function, and write the domain in interval notation.

$$p(x) = \frac{\ln(\sqrt{3x - 1})}{|3x - 1|}$$

$$q(x) = \arctan(2x + 3\pi) - \arcsin(2x + 3\pi)$$

You must clearly and coherently justify your work. You cannot provide only the final answer. Circle your final answer for each function. No decimal number as a final answer.

5 Question No.5 - Challenging (4 points - Part (A) is worth 1 point; Part (B) is worth 1 point; Part (C) is worth 2 points)

Version 1

The calculation below has an error.

$$3 > 2 \quad (1)$$

$$3 \ln\left(\frac{1}{2}\right) > 2 \ln\left(\frac{1}{2}\right) \quad (2)$$

$$\ln\left(\frac{1}{2}\right)^3 > \ln\left(\frac{1}{2}\right)^2 \quad (3)$$

$$\ln\left(\frac{1}{8}\right) > \ln\left(\frac{1}{4}\right) \quad (4)$$

$$\frac{1}{8} > \frac{1}{4} \quad (5)$$

Complete each part:

Part (A) Identify the line, by its number, which contains the error.

Part (B) State the mathematical statement or rule that the line which you identified in Part (A) uses incorrectly.

Part (C) Starting with the line containing the error, modify the calculation to make it correct. You must clearly and coherently justify your work.

Version 2

The calculation below has an error.

Solve the equation $\sqrt{(x-1)^2 + 4x} = 3x + 4$ for x .

$$\sqrt{(x-1)^2 + 4x} = 3x + 4 \quad (1)$$

$$\sqrt{(x+1)^2} = 3x + 4 \quad (2)$$

$$x + 1 = 3x + 4 \quad (3)$$

$$x = -\frac{3}{2} \quad (4)$$

Complete each part:

Part (A) Identify the line, by its number, which contains the error.

Part (B) State the mathematical statement or rule that the line which you identified in Part (A) uses incorrectly.

Part (C) Starting with the line containing the error, modify the calculation to make it correct. You must clearly and coherently justify your work.

Version 3

The calculation below has an error.

Solve the equation $\ln(x^2 - 1) + \ln(x - 2) = \ln(2) + \ln(1 - x)$ for x .

$$\ln(x^2 - 1) + \ln(x - 2) = \ln(2) + \ln(1 - x) \quad (1)$$

$$\ln(x^3 - 2x^2 - x + 2) = \ln(2 - 2x) \quad (2)$$

$$x^3 - 2x^2 + x = 0 \quad (3)$$

$$x(x - 1)^2 = 0 \quad (4)$$

$$\text{The solutions are } x = 0 \text{ and } x = 1. \quad (5)$$

Complete each part:

Part (A) Identify the line, by its number, which contains the error.

Part (B) State the mathematical statement or rule that the line which you identified in Part (A) uses incorrectly.

Part (C) Starting with the line containing the error, modify the calculation to make it correct. You must clearly and coherently justify your work.

Version 4

The calculation below has an error.

Solve the equation $\sqrt{2x + 4} = \sqrt{6x + 1} - 1$ for x .

$$\sqrt{2x + 4} = \sqrt{6x + 1} - 1 \quad (1)$$

$$(\sqrt{2x + 4})^2 = (\sqrt{6x + 1} - 1)^2 \quad (2)$$

$$2\sqrt{6x + 1} = 4x - 2 \quad (3)$$

$$6x + 1 = 4x^2 - 4x + 1 \quad (4)$$

$$2x(2x - 5) = 0 \quad (5)$$

$$\text{The solutions are } x = 0 \text{ and } x = \frac{5}{2}. \quad (6)$$

Complete each part:

Part (A) Identify the line, by its number, which contains the error.

Part (B) State the mathematical statement or rule that the line which you identified in Part (A) uses incorrectly.

Part (C) Starting with the line containing the error, modify the calculation to make it correct. You must clearly and coherently justify your work.