
MAT132H5 - Differential Calculus for Life Sciences
Term Test 1 - October 4, 2019 - Version ARKP

Aids Permitted: None

Time Allotted: 110 minutes

Instructions

- Please have your student card ready for inspection, turn off all cellular phones, and read all the instructions carefully.
- This test contains two parts: Part A (36 marks) contains **nine** short questions, and Part B (64 marks) contains **four** questions. All answers are to be given in this booklet.
- Please **do not write anything on the QR codes** that appear at the top of each page.
- Check that this test has **12 pages**, including this cover page.
- There is a formula sheet on page 11, and space for rough work. You can also use page 10 and 12 for rough work. You may tear off the formula sheet, but you must **submit it** together with the rest of the test.
- Make sure to provide **exact answers**, using symbols such as $\sqrt{\quad}$, e and π , if needed.

GOOD LUCK!

PART A (36 marks)

In this part, **clearly indicate your final short answer in the appropriate box.**

You must show your work (if any), even though **only the final answer will be graded.**

Simplify your answers **as much as possible.** Each question is worth **4 marks.**

1. Solve the inequality $|3x - 4| - 5 \geq 0$. Provide your answer using **the interval notation.**

Answer for 1.

2. Find the **natural domain** and the **range** of the function $h(x) = \sqrt{x+1} - 5$.

Provide your answer using **the interval notation.**

Answer for 2.

Domain: _____ ; Range: _____

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3. If $f(x) = \cos x$ and $g(x) = \frac{2x}{3}$, find $f \circ g\left(\frac{\pi}{4}\right)$ and $g \circ f\left(\frac{\pi}{4}\right)$ (note that \circ denotes **composition**).
Simplify your answers, so that they **do not** include any trigonometric functions.

Answer for 3.

$$f \circ g\left(\frac{\pi}{4}\right) = \qquad g \circ f\left(\frac{\pi}{4}\right) =$$

4. **Calculate** the value of $2 \log_8(3) - \log_8(18)$.

Your answer should be a **single number**, that **does not** include any logarithms.

Answer for 4.

5. The function $p(x) = \sin^{-1}(e^x - 2)$ is **one-to-one** on its domain. Find an expression for **its inverse** $p^{-1}(x)$.

Answer for 5.

$$p^{-1}(x) =$$

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6. Calculate the value of $\frac{9^{2/3} \cdot 8^{5/6}}{3^{1/3} \cdot 2^{1/2}}$. Your answer should be a **single integer**.

Answer for 6.

7. **Evaluate** the limit $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^2 + x - 2}$.

Your answer should be a **number**, or **DNE** if the limit does not exist.

Answer for 7.

8. **Evaluate** the limit $\lim_{x \rightarrow 0^-} \frac{\sin(8x)}{2 \cdot |x|}$.

Your answer should be a **number**, or **DNE** if the limit does not exist.

Answer for 8.

9. Suppose that a function f , with domain \mathbb{R} , satisfies $\lim_{x \rightarrow 3^-} f(x) = f(3) + \frac{1}{2}$ and $\lim_{x \rightarrow 3^+} f(x) = f(3) - \frac{1}{2}$.

Is f continuous at $x=3$? **Circle the correct answer in the box.**

Answer for 9.

- A. Yes, f is continuous at $x=3$.
 - B. No, f has a removable discontinuity at $x=3$.
 - C. No, f has a jump discontinuity at $x=3$.
 - D. No, f has an infinite discontinuity at $x=3$.
 - E. No, f has an oscillating discontinuity at $x=3$.

PART B (64 marks)

In this part you are required to provide **full solutions** and to **show all your work**.

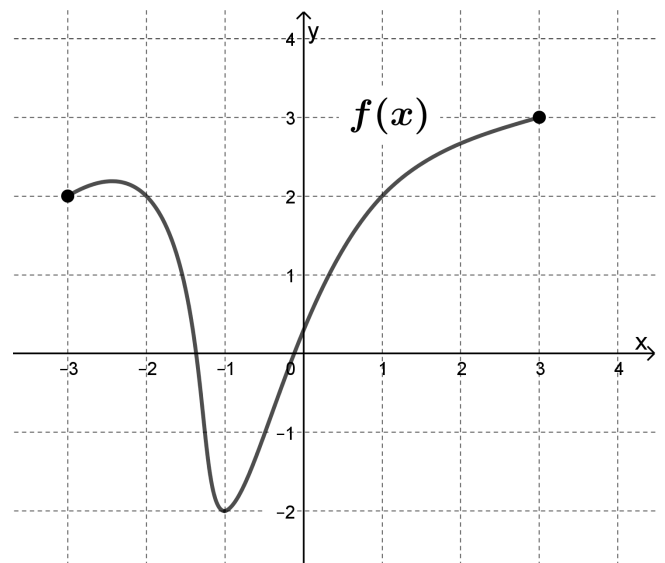
A correct answer obtained with false reasoning or with no reasoning will not receive any marks.

Each question is worth **16 marks**.

1. Here is a graph of a function f . **Answer the questions below.**

No explanation is needed for parts (a) and (b).

(a) What are the **domain** and **range** of f ?



(b) For which x value(s) is $f(x) = 2$?

(c) Is f a **one-to-one** function? **Explain.**

(d) **How many solutions** does the equation $f(x) = x$ have? **Explain.**

2. Consider the function $g(x) = x \cdot \tan x$.

(a) Is the function $g(x)$ **even, odd or neither**? **Explain**.

(b) We create a **new function** $h(x)$ as follows: First, we stretch the graph of $g(x)$ horizontally by a factor of 3. Then, we reflect it about the x-axis. Finally, we shift the graph 2 units upwards.

Find an explicit formula for $h(x)$. **Explain** your answer.

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3. Suppose that a function f satisfies $\cos x + 1 \leq f(x) \leq \frac{(\sqrt{x} + 1)^2 - 1}{\sqrt{x}}$ for all $x > 0$.

Use a theorem discussed in class to **find the value** of the limit $\lim_{x \rightarrow 0^+} f(x)$, or show that it does not exist.

4. Use a theorem discussed in class to show that the equation $3^x - x^3 - x = 0$ has **at least two solutions** in the interval $[1, 4]$. Write a **detailed solution** and show all your work.

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DO NOT TEAR OFF THIS PAGE!

FORMULA SHEET

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

SPACE FOR ROUGH WORK

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