The Robert Gillespie ACADEMIC SKILLS CENTRE

# Sequences

## Sequences

A sequence is a list of numbers written in a definite order denoted as:

$$a_1, a_2, \ldots, a_n, \ldots = \{a_n\},$$

where $a_1$ is the first term of the sequence, $a_2$ is the second term and in general, $a_n$ is the nth term. (Note that often, unless stated otherwise, it is assume that the values of $n$ start with $n = 1$).

For example, $5, 10, 15, 20, 25\ldots = \{5n\} = \{a_n\}$ or

$$-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}\ldots = \left\{\frac{1}{(-3)^n}\right\}_{n=1}^{\infty}$$

If a sequence $\{a_n\}$ has a limit, that is $\lim_{n\to\infty} a_n = L$ where $L$ is a real number, then it is said to be convergent, and converges to $L$.

If the limit of the sequence does not exist, then the sequence is said to be divergent.

For instance, the sequence $\{r^n\}$ is convergent if $-1 < r \le 1$, that is,

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & , \quad \text{if } -1 < r < 1 \\ 1 & , \quad \text{if } r = 1 \end{cases}$$

, and divergent otherwise.

Example. Determine if each sequence is convergent or divergent.

(a) $\{a_n\} = \{3 + e^{-n}\}$ (b) $\{a_n\} = \{(-0.5)^n\}$

(c) $\{a_n\} = \left\{ \frac{n^2}{3+5n} \right\}$ (d) $\{a_n\} = \left\{ (-1)^n \right\}$

Solution. (a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} (3 + e^{-n}) = 3$. Convergent.

(b) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-0.5)^n = 0$. Convergent.

(c) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2}{3+5n} = \lim_{n\to\infty} \frac{n}{3/n+5} = +\infty$. Divergent.

(d) Note that $\{a_n\} = \left\{ (-1)^n \right\} = -1, 1, -1, 1, \ldots$ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n$ does not exist. Divergent.

.