The Robert Gillespie ACADEMIC SKILLS CENTRE

Integration by Parts

Formula for Integration by Parts

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$$

or $$\int u\,dv = uv - \int v\,du$$

How to pick $f(x)$ and $g'(x)$, i.e., u and dv?

Here is a suggestion!

Remember the word LATE:

L – logarithmic ex. $\ln(x)$
A – algebraic ex. $x^2 + x$
T – trigonometric ex. $\sin(x)$
E – exponential ex. e^x

The first expression that appears in the word LATE in the integrand will be u, and the rest will be dv.

Take the derivative of u and integrate dv. Then, use the formula for integration by parts.

Example. Find $\int xe^{2x}dx$.

Solution. According to LATE, $u = x$ and $dv = e^{2x}dx$.

$$u = x \qquad \int dv = \int e^{2x}\,dx$$

and

$$du = dx \qquad v = \frac{e^{2x}}{2}$$

$$\int xe^{2x}\,dx = (x)\left(\frac{e^{2x}}{2}\right) - \int \frac{e^{2x}}{2}dx = \frac{xe^{2x}}{2} - \frac{1}{2}\left(\frac{e^{2x}}{2}\right) + C$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C = \frac{e^{2x}}{2}\left(x - \frac{1}{2}\right) + C$$

Definite Integration by Parts

The integration by parts formula can be applied to definite integrals by noting that

$$\int_b^a u\,dv = uv\Big|_a^b - \int_b^a v\,du.$$

Computing a Definite Integral by Integration by Parts

Step 1. Solve the integral as an indefinite integral.

Step 2. Evaulate it over the interval of integration.

Example. Find $\int_1^e \frac{\ln x}{x^2}dx$.

Solution. Step 1. According to LATE,

$u = \ln x$ and $dv = \frac{1}{x^2}dx = x^{-2}dx$.

$$u = \ln x \qquad \int dv = \int \frac{1}{x^2}dx = \int x^{-2}\,dx$$

$$du = \frac{1}{x}dx \quad \text{and} \quad v = \frac{-1}{x} \quad .$$

$$\int \frac{\ln x}{x^2}dx = (\ln x)\left(\frac{-1}{x}\right) - \int \left(\frac{-1}{x}\right)\left(\frac{1}{x}dx\right)$$

$$= -\frac{\ln x}{x} - \int x^{-2}\,dx$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

Step 2.

$$\int_1^e \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x}\Bigg|_1^e$$

$$= \left(-\frac{\ln e}{e} - \frac{1}{e}\right) - \left(-\frac{\ln 1}{1} - \frac{1}{1}\right)$$

$$= -\frac{2}{e} + 1$$