# Trade Wars along the Global Value Chains

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**Abstract**

We propose a multi-sector, multi-factor Krugman model with production linkages to quantify the consequences of the recent US-China trade conflicts. The model emphasizes the importance of high-tech or strategic sectors that (i) are skilled-intensive, (ii) utilize skilled-intensive inputs, (iii) have low elasticities of substitution across varieties within sector, and (iv) are not or little substitutable in producing other products. Semiconductor industry, one example of these strategic sectors, has drawn substantial attention in the recent trade conflicts. Our quantification suggests that these strategic industries could suffer disproportionately from trade conflicts. Considering the role of strategic industries in the global value chains, we find that the proposed US-China tariff war would decrease the U.S. real income by 0.43 percent and the China's real income by 0.36 percent.

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# 1 Model

Consider a world with $N$ countries, indexed by $i$ and $n$, with a mass $L_i$ unskilled labor and $S_i$ skilled labor in each $i$. There are $J$ sectors, indexed by $j$ and $s$. Each sector $j$ in country $i$ is endowed with a unit mass of firms. The representative consumer in country $i$ has Cobb-Douglas preferences over nontradable final goods from all sectors, with $\alpha_i^j$ being the expenditure shares on goods $j$. Final goods are aggregated from tradable intermediates and also used as materials. Intermediate goods are produced by firms under monopolistic competition using unskilled labor, skilled labor, and materials. The firm producing goods $j$ in country $i$ is endowed with the total factor productivity $T_i^j$, which is common across all firms in sector $j$ of country $i$.

Traded intermediate goods are subject to two types of bilateral trade costs. First, there is an iceberg trade cost $\tau_{in}^j$ of shipping goods from $i$ to $n$, with $\tau_{ii}^j = 1$. Second, there is *ad valorem* tariff $t_{in}^j$ imposed by importing country $n$ on goods $j$ from country $i$, with $t_{ii}^j = 0$.

## 1.1 Final goods production

Final goods in sector $j$ are aggregated from intermediate goods by a CES function with the elasticity of substitution $\sigma_j > 1$.

## 1.2 Intermediates production

Each variety of intermediates $j$ is produced by a firm using unskilled labor, skilled labor, and materials. The unit cost of variety $\omega$ of intermediate $j$ in country $i$ is

$$c_i^j(\omega) = c_i^j = \frac{1}{T_i^j}\left[w_i^{1-\psi_j} + \left(\frac{h_i}{A_i^j}\right)^{1-\psi_j}\right]^{\frac{\beta_i^j}{1-\psi_j}} \underbrace{\left(\sum_{g\in G_i^j}\left[\sum_{s\in g}\gamma_i^{sj}\left(P_i^s\right)^{1-\eta_i^{gj}}\right]^{\frac{1-\mu_i^j}{1-\eta_i^{gj}}}\right)^{\frac{1-\beta_i^j}{1-\mu_i^j}}}_{\text{Nested-CES sectoral price aggregator}}, \tag{1}$$

where $w_i$ is the wage for unskilled labor, $h_i$ is the wage for skilled labor, $A_i^j$ is the skill-biased productivity for goods $j$ in country $i$. $\psi_j$ is the elasticity of substitution between unskilled labor and capital in sector $j$. Notably, "$gj$" refers to group $g$ of sectors for producing good $j$. $\eta_i^{gj}$ is the elasticity of substitution within group $g$ for producing good $j$ in country $i$, whereas $\mu_i^j$ is the elasticity of substitution across groups for producing good $j$ in country $i$.

### 1.3 Equilibrium

In this subsection, we characterize the aggregate economy and define the equilibrium. Then the final price index for good $j$ in country $i$ can be expressed as

$$P_n^j = \left[\sum_{i=1}^{N} \tilde{\sigma}_j \left[c_i^j \tau_{in}^j \left(1+t_{in}^j\right)\right]^{1-\sigma_j}\right]^{\frac{1}{1-\sigma_j}}, \quad \tilde{\sigma}_j = \frac{\sigma_j}{\sigma_j - 1}. \tag{2}$$

Let $X_n^j$ be the total expenditure on sector $j$ in country $n$. Then the sales of good $j$ from country $i$ in country $n$ can be given by

$$X_{in}^j = \frac{\tilde{\sigma}_j \left[c_i^j \tau_{in}^j \left(1+t_{in}^j\right)\right]^{1-\sigma_j}}{\left(P_n^j\right)^{1-\sigma_j}} X_n^j. \tag{3}$$

The wage income for unskilled labor is then

$$w_i L_i = \sum_{j=1}^{J} \left(1 - \frac{1}{\sigma_j}\right) \beta_i^j \frac{w_i^{1-\psi_j}}{w_i^{1-\psi_j} + \left(\frac{h_i}{A_i^j}\right)^{1-\psi_j}} \sum_{n=1}^{N} \frac{1}{1+t_{in}^j} X_{in}^j. \tag{4}$$

The wage income for skilled labor is given by

$$h_i S_i = \sum_{j=1}^{J} \left(1 - \frac{1}{\sigma_j}\right) \beta_i^j \frac{\left(\frac{h_i}{A_i^j}\right)^{1-\psi_j}}{w_i^{1-\psi_j} + \left(\frac{h_i}{A_i^j}\right)^{1-\psi_j}} \sum_{n=1}^{N} \frac{1}{1+t_{in}^j} X_{in}^j. \tag{5}$$

The total income is the sum of wage incomes, profits, and tariff revenue:

$$Y_i = w_i L_i + h_i S_i + \sum_{j=1}^{J} \frac{1}{\sigma_j} \sum_{n=1}^{N} \frac{1}{1+t_{in}^j} X_{in}^j + \sum_{j=1}^{J} \sum_{k=1}^{N} \frac{t_{ki}^j}{1+t_{ki}^j} X_{ki}^j. \tag{6}$$

Sectoral expenditure consists of final consumption and intermediates expenditure:

$$X_i^j = \alpha_i^j Y_i +$$

$$\sum_{s=1}^{J} \left(1 - \frac{1}{\sigma_j}\right) \left(1 - \beta_i^s\right) \frac{\gamma_i^{js} \left(P_i^j\right)^{1-\eta_i^{gs}}}{\sum_{j' \in g} \gamma_i^{j's} \left(P_i^{j'}\right)^{1-\eta_i^{gs}}} \frac{\left[\sum_{j' \in g} \gamma_i^{j's} \left(P_i^{j'}\right)^{1-\eta_i^{gs}}\right]^{\frac{1-\mu_i^s}{1-\eta_i^{gs}}}}{\sum_{g' \in G_i^s} \left[\sum_{j' \in g'} \gamma_i^{j's} \left(P_i^{j'}\right)^{1-\eta_i^{g's}}\right]^{\frac{1-\mu_i^s}{1-\eta_i^{g's}}}} \sum_{n=1}^{N} \frac{X_{in}^s}{1+t_{in}^s}. \tag{7}$$

# 2 Quantification

## 2.1 "Exact-Hat" Algebra

To quantify the effects of trade wars, we need to compute the changes in equilibrium outcomes with respect to the changes in tariff rates. We denote $x'$ as the level after changes for any variable $x$ and $\hat{x} := \frac{x'}{x}$. Moreover, we denote $\pi_{in}^j := \frac{X_{in}^j}{X_n^j}$.

First, changes in unit costs can be expressed as

$$\hat{c}_i^j = \left[(1-\lambda_i^j)\hat{w}_i^{1-\psi_j} + \lambda_i^j \hat{h}_i^{1-\psi_j}\right]^{\frac{\beta_i^j}{1-\psi_j}} \left(\sum_{g \in G_i^j} \delta_i^{gj} \left[\sum_{s \in g} \chi_i^{sj} \left(\hat{P}_i^s\right)^{1-\eta_i^{gj}}\right]^{\frac{1-\mu_i^j}{1-\eta_i^{gj}}}\right)^{\frac{1-\beta_i^j}{1-\mu_i^j}}, \tag{8}$$

where $\lambda_i^j$ is the wage income share of skilled workers in sector $j$ and country $i$, $\chi_i^{sj}$ is the expenditure share of good $s$ in intermediates group $g$ that produces good $j$ in country $i$, and $\delta_i^{gj}$ is the expenditure share of intermediates group $g$ in producing good $j$ in country $i$.

Changes in trade share can be given by

$$\hat{\pi}_{in}^j = \frac{\left[\hat{c}_i^j \widehat{1+t_{in}^j}\right]^{1-\sigma_j}}{\left(\hat{P}_n^j\right)^{1-\sigma_j}}. \tag{9}$$

Changes in price indices can be given by

$$\hat{P}_n^j = \left[\sum_{i=1}^N \pi_{in}^j \left[\hat{c}_i^j \widehat{1+t_{in}^j}\right]^{1-\sigma_j}\right]^{\frac{1}{1-\sigma_j}}. \tag{10}$$

Changes in the wage of unskilled workers can be expressed as

$$\hat{w}_i w_i L_i = \sum_{j=1}^J \left(1-\frac{1}{\sigma_j}\right)\beta_i^j \frac{(1-\lambda_i^j)\hat{w}_i^{1-\psi_j}}{(1-\lambda_i^j)\hat{w}_i^{1-\psi_j} + \lambda_i^j \hat{h}_i^{1-\psi_j}} \sum_{n=1}^N \frac{1}{\widehat{1+t_{in}^j}} \frac{1}{1+t_{in}^j} \hat{\pi}_{in}^j \hat{X}_n^j X_{in}^j. \tag{11}$$

Changes in the wage of skilled workers can be expressed as

$$\hat{h}_i h_i S_i = \sum_{j=1}^J \left(1-\frac{1}{\sigma_j}\right)\beta_i^j \frac{\lambda_i^j \hat{h}_i^{1-\psi_j}}{(1-\lambda_i^j)\hat{w}_i^{1-\psi_j} + \lambda_i^j \hat{h}_i^{1-\psi_j}} \sum_{n=1}^N \frac{1}{\widehat{1+t_{in}^j}} \frac{1}{1+t_{in}^j} \hat{\pi}_{in}^j \hat{X}_n^j X_{in}^j. \tag{12}$$

Changes in the total income can be given by

$$\hat{Y}_i Y_i = \hat{w}_i w_i L_i + \hat{h}_i h_i S_i + \sum_{j=1}^{J} \frac{1}{\sigma_j} \sum_{n=1}^{N} \frac{1}{\widehat{1+t_{in}^j}} \frac{1}{1+t_{in}^j} \hat{\pi}_{in}^j \hat{X}_n^j X_{in}^j + \sum_{j=1}^{J} \sum_{k=1}^{N} \left( \frac{t_{ki}^j}{1+t_{ki}^j} \right)' \hat{\pi}_{ki}^j \hat{X}_i^j X_{ki}^j. \tag{13}$$

Finally, changes in sectoral expenditure can be given by

$$\hat{X}_i^j X_i^j = \alpha_i^j \hat{Y}_i Y_i +$$

$$\sum_{s=1}^{J} \left(1 - \frac{1}{\sigma_j}\right) (1-\beta_i^s) \frac{\chi_i^{js} \left(\hat{P}_i^j\right)^{1-\eta_i^{gs}}}{\sum_{j' \in g} \chi_i^{j's} \left(\hat{P}_i^{j'}\right)^{1-\eta_i^{gs}}} \frac{\delta_i^{gs} \left[\sum_{j' \in g} \chi_i^{j's} \left(\hat{P}_i^{j'}\right)^{1-\eta_i^{gs}}\right]^{\frac{1-\mu_i^s}{1-\eta_i^{gs}}}}{\sum_{g' \in G_i^s} \delta_i^{g's} \left[\sum_{j' \in g'} \chi_i^{j's} \left(\hat{P}_i^{j'}\right)^{1-\eta_i^{g's}}\right]^{\frac{1-\mu_i^s}{1-\eta_i^{g's}}}} \sum_{n=1}^{N} \frac{(X_{in}^s)'}{1+(t_{in}^s)'}. \tag{14}$$

To conduct exact-hat algebra in this model, we need data on bilateral trade shares $\left(\pi_{in}^j\right)$, sectoral expenditure $(X_n^j)$, input expenditure shares $\tilde{\gamma}_i^{js}$, the wage income share of skilled workers $\left(\lambda_i^j\right)$, and the tariff rates $\left(t_{in}^j\right)$. We also need the values of $\left(\sigma_j, \psi_j, \alpha_i^j, \beta_i^j, \mu_i^j, \eta_i^{gj}\right)$.

## 2.2 Parameterization of the IO Linkages

Our nested CES IO linkages are very flexible in capturing the substitutability across sectors. However, in practice, it is challenging to estimate a large number of elasticities of substitution within and across groups. Therefore, in our baseline quantification practice, we simplify our parameterization as follows.

In each country we classify sectors as strategic ("S") and non-strategic ("NS"). Within each group, we assume that sectors are aggregated by Cobb-Douglas production function, i.e. $\eta_i^{gj} = 1$. Across groups, instead, we assume that the production function is Leontief, i.e. $\mu_i^j = 0$.

Under this parameterization, Equation (8) can be simplified into

$$\hat{c}_i^j = \left[(1-\lambda_i^j)\hat{w}_i^{1-\psi_j} + \lambda_i^j \hat{h}_i^{1-\psi_j}\right]^{\frac{\beta_i^j}{1-\psi_j}} \left[\sum_{g \in G_i^j} \delta_i^{gj} \prod_{s \in g} \left(\hat{P}_i^s\right)^{\chi_i^{sj}}\right]^{1-\beta_i^j}. \tag{15}$$

Accordingly, Equation (14) can be expressed as

$$\hat{X}_i^j X_i^j = \alpha_i^j \hat{Y}_i Y_i + \sum_{s=1}^{J} \left(1 - \frac{1}{\sigma_j}\right) \left(1 - \beta_i^s\right) \chi_i^{js} \frac{\delta_i^{gs} \left[\prod_{j' \in g} \left(\hat{P}_i^{j'}\right)^{\chi_i^{j's}}\right]}{\sum_{g' \in G_i^s} \delta_i^{g's} \left[\prod_{j' \in g'} \left(\hat{P}_i^{j'}\right)^{\chi_i^{j's}}\right]} \sum_{n=1}^{N} \frac{\left(X_{in}^s\right)'}{1 + \left(t_{in}^s\right)'}. \tag{16}$$