Implicit Differentiation

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Explicit equation (i.e., y given explicitly as a function of x):

$$y = e^{3x} - x^2 + \ln x$$

Explicit equation (i.e., y given explicitly as a function of x, or x given explicitly as a function of y):

$$x^2y + y^2 = e^{xy} + \sin(x + y)$$

For functions given IMPLICITLY:

<u>Step 1.</u> Assuming that y is a function of x, i.e., $y = f(x)$, take the usual derivative with respect to x, keeping in mind to use the chain rule when differentiating terms involving y (which, of course, involves multiplying by y' or $\dfrac{dy}{dx}$).

<u>Step 2.</u> Solve for y', if possible, or requested.

Example. Differentiate the following equations.

(a) $2x^3 - 3xy^2 + y^4 = -20$

(b) $xy = e^{-xy}$

Solution. (a) Step 1.

$$2x^3 - 3xy^2 + y^4 = -20$$
$$6x^2 - 3[y^2 + 2yy'x] + 4y^3y' = 0$$
$$6x^2 - 3y^2 - 6yy'x + 4y^3y' = 0$$

Step 2.

$$\begin{aligned} -6xyy' + 4y^3y' &= -6x^2 + 3y^2 \\ y'(-6xy + 4y^3) &= -6x^2 + 3y^2 \quad , \\ y' &= \frac{-6x^2 + 3y^2}{-6xy + 4y^3} \end{aligned}$$

(b) Step 1.

$$\begin{aligned} xy &= e^{-xy} \\ y + xy' &= e^{-xy}(-y - xy') \\ y + xy' &= -e^{-xy}y - e^{-xy}xy' \end{aligned}$$

Step 2.

$$\begin{aligned} xy' + e^{-xy}xy' &= -e^{-xy}y - y \\ y'(x + e^{-xy}x) &= -y(e^{-xy} + 1) \quad , \\ y' &= \frac{-y(e^{-xy} + 1)}{x(e^{-xy} + 1)} = \frac{-y}{x} \end{aligned}$$

since $e^{-xy} + 1 \neq 0$.